

## Polynomial and Rational Inequalities

### Steps for Solving Polynomial and Rational Inequalities

1. Write the inequality so that a polynomial or rational expression  $f$  is on the left side and zero is on the right side in one of the following forms:

$$f(x) > 0$$

$$f(x) < 0$$

$$f(x) \geq 0$$

$$f(x) \leq 0$$

For rational expressions, be sure that the left side is written as a *single quotient* and find its domain.

2. Determine the real numbers at which the expression  $f$  on the left side equals zero and, if the expression is rational, the real numbers at which the expression  $f$  on the left side is undefined. (In other words, find any  $x$ -intercepts and vertical asymptotes of the graph of  $y = f(x)$ ).
3. Use the numbers found in Step 2 to separate the  $x$ -axis into intervals.
4. Determine whether the graph of  $y = f(x)$  is above or below the  $x$ -axis in each interval.

The solutions of  $f(x) > 0$  are the  $x$ -values for which the graph is **above** the  $x$ -axis.

The solutions of  $f(x) \geq 0$  are the  $x$ -values for which the graph is **on or above** the  $x$ -axis.

The solutions of  $f(x) < 0$  are the  $x$ -values for which the graph is **below** the  $x$ -axis.

The solutions of  $f(x) \leq 0$  are the  $x$ -values for which the graph is **on or below** the  $x$ -axis.

★ **Warning:** For  $f(x) \geq 0$  and  $f(x) \leq 0$ , include the  $x$ -intercepts *but not the asymptotes* in the solution set.

-OR-

4. Select a number in each interval and evaluate  $f(x)$  at the number.
  - a. If the value of  $f(x)$  is positive, then  $f(x) > 0$  for all numbers  $x$  in the interval.
  - b. If the value of  $f(x)$  is negative, then  $f(x) < 0$  for all numbers  $x$  in the interval.

The solution set is all the intervals that make the inequality true.

★ **Warning:** For  $f(x) \geq 0$  and  $f(x) \leq 0$ , include the values for which  $f(x) = 0$  (the  $x$ -intercepts) *but not the values for which  $f(x)$  is undefined* (the asymptotes) in the solution set.

**Example:** Solve the inequality  $x^4 > 9x^2$ , and graph the solution set.

**Example:** Solve the inequality  $x^4 \geq 8x$ , and graph the solution set.

**Example:** Solve the inequality  $\frac{(x+2)(4-x)}{(x+1)^2} < 0$ , and graph the solution set.

**Example:** Solve the inequality  $\frac{3x+5}{x+4} \leq 2$ , and graph the solution set.